CHAPTER 50

Queueing Representation of Kinematic Waves

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50.1 Introduction

MATSim comes with a number of mobsims (cf. Sections 4.3, 43.1); the most important are the so-called QSim and JDEQSim. These differ from the implementation perspective (time-stepping vs. event-based, degree of parallelism), but all are (at least approximate) solvers of the same underlying traffic flow model. The purpose of this chapter is to relate MATSim's mobsims to the existing traffic flow theory. There are other simulation packages rooted in the same underlying modeling concepts (Tian et al., 2007; Zhou and Taylor, 2014).

The flow-density relationship (also called FD (Fundamental Diagram)) shown in Figure 50.1 is at the heart of MATSim's traffic flow model. Given a long, homogeneous road, it predicts average flow q (in vehicles per time unit) through any cross-section of that road, given an average vehicle density ϱ (in vehicles per length unit) on that road.

The FD is defined as the minimum of a sending function $S(\varrho)$ (solid) and a receiving function $R(\varrho)$ (dashed), resulting overall in a triangular curve parametrized by free flow speed v, maximum density $\hat{\varrho}$ and backward wave speed w. The maximum velocity is an observable parameter that can be set in the network file (`freespeed` attribute of the `link` element). The maximum density equals one over the length of a vehicle (`effectivecellsize` attribute of the `links` element) for a single-lane link and needs to be multiplied with the number of lanes (`permlanes` attribute of the `link` element), otherwise. The backward wave speed turns out to be the (negative of the) ratio of vehicle length to the safety time gap adopted by drivers in congested conditions. This parameter is fairly constant; a vehicle length of 7.5 meters and a time gap of 2 seconds leads to a value of (minus) 13.5 kilometers per hour. The backward wave speed can be set in the JDEQSim through the `gapTravelSpeed` parameter; it cannot currently be set in the QSim.

The considered FD alone applies only in stationary conditions, where it predicts that (i) flow increases linearly with density at low densities (i.e., in uncongested conditions); (ii) flow decreases

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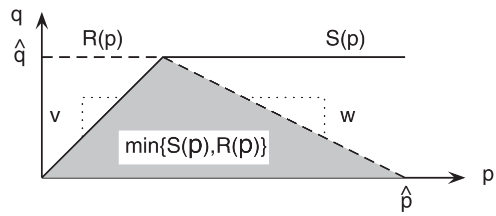

Figure 50.1: Fundamental diagram.

linearly with density at high densities (i.e., in congested conditions); and (iii) in between, it attains a maximal value constituting the flow capacity

$$\hat{q} = \frac{vw\hat{\varrho}}{v+w} \tag{50.1}$$

of the link. This parameter represents the maximum throughput of the link in the absence of any other flow constraint (such as downstream traffic lights or other bottlenecks, which are discussed further below).

A realistic representation of non-stationary traffic flow (where density and flow change over space and time) is possible by inserting the FD into a continuity equation (which intuitively models vehicle conservation, in the sense that vehicles cannot vanish or spontaneously appear on a road segment without on- and off-ramps). This leads to the KWM (Kinematic Wave Model) of traffic flow (Lighthill and Witham, 1955; Richards, 1956), where the sending and receiving function receive an intuitive interpretation: The instantaneous flow across any interface, possibly with different densities prevailing and FDs applying up- and downstream of that interface, is defined by (i) inserting the density upstream of the interface into the upstream sending function, (ii) inserting the density downstream of that interface into the downstream receiving function and (iii) taking the minimum of these two quantities (Daganzo, 1994; Lebacque, 1996). Intuitively: The flow is limited by what can be sent from upstream and what can be received downstream, but otherwise it is maximized.

The remainder of this chapter expresses MATSim's link model (Section 50.2) and its node model (Section 50.3) in terms of the sending and receiving function framework of the KWM. Some technical detail is omitted from the presentation for the sake of readability; pointers to the literature are provided.

50.2 Link Model

To compute flows entering and leaving a link, one needs to know how much flow can maximally enter the link and how much flow can maximally leave the link. Both constraints depend on the internal (congestion) state of the link. In symbols, one is interested in the instantaneous receiving flow rate R of the link's upstream end and the instantaneous sending flow rate S of the link's downstream end. Multiplying these rates by the duration δ of a simulation time step then yields the maximum number of vehicles that can enter or leave the link during a time step.

MATSim also needs to compute these quantities; how it does so is rooted in Newell's "simplified theory of kinematic waves" (Newell, 1993), which provides a tracktable recipe for computing flow and density anywhere in a link, given that one keeps track only of the flows at the link's up- and downstream interface. In the continuum model (i.e., one that allows for real-valued flows and densities at real-valued locations and times) specified by Newell (1993), the cumulative in- and outflow of a link are defined as

$$N^{\text{in}}(t) = \int_0^t q^{\text{in}}(z)dz \tag{50.2}$$

$$N^{\text{out}}(t) = \int_0^t q^{\text{out}}(z)dz \tag{50.3}$$

where t denotes time, q^{in} and $q^{\text{out}}$are the instantaneous in- and outflow rates (in vehicles per time unit) of the link and an initially (at $t = 0$) empty link is assumed. From MATSim's vehicle-discrete perspective, cumulative inflow (outflow) at a given point in time hence represents the total number of vehicles having entered (left) the link up to that point in time.

Yperman et al. (2006); Yperman (2007) observe that if Newell's theory allows computation of instantaneous densities anywhere in a link, then it also allows computation of densities at the up- and downstream ends of that link. Inserting these densities in the link's sending and receiving function then allows expressing the sending and receiving flows as functions of time-shifted cumulative in- and outflows only, with the time-shifts specified according to the original Newell (1993) formula:

$$R(t) = \min\left\{\hat{\varrho}L - \left[N^{\text{in}}(t) - N^{\text{out}}(t + \delta - L/|w|)\right], \hat{q}\delta\right\} \tag{50.4}$$

$$S(t) = \min\left\{\left[N^{\text{in}}(t + \delta - L/v) - N^{\text{out}}(t)\right], \hat{q}\delta\right\} \tag{50.5}$$

where L is the link length and δ is the (small) discrete time step length. Yperman (2007) provides some intuition for this rather formal specification.

The connection to MATSim can now be made explicit by labeling the two bracketed terms in Equation (50.4) and Equation (50.5) as "upstream queue" (UQ) and "downstream queue" (DQ) (Osorio et al., 2011; Osorio and Flötteröd, published online in *Articles in Advance*):

$$\text{UQ}(t) = N^{\text{in}}(t) - N^{\text{out}}(t + \delta - L/|w|) \tag{50.6}$$

$$\text{DQ}(t) = N^{\text{in}}(0, t + \delta - L/v) - N^{\text{out}}(t). \tag{50.7}$$

These expressions can be given a recursive meaning. Evaluating $\text{UQ}(t) - \text{UQ}(t - \delta)$ yields $[N^{\text{in}}(t) - N^{\text{in}}(t - \delta)] - [N^{\text{out}}(t + \delta - L/|w|) - N^{\text{out}}(t - L/|w|)]$, which under the assumption that flow rates are held constant throughout a simulation time step simplifies into $\delta[q^{\text{in}}(t - \delta) - q^{\text{out}}(t - L/|w|)]$. From this (and symmetric operations for DQ), one obtains

$$\text{UQ}(t) = \text{UQ}(t - \delta) + \delta\left[q^{\text{in}}(t - \delta) - q^{\text{out}}(t - L/|w|)\right] \tag{50.8}$$

$$\text{DQ}(t) = \text{DQ}(t - \delta) + \delta\left[q^{\text{in}}(t - L/v) - q^{\text{out}}(t - \delta)\right]. \tag{50.9}$$

These recursive definitions turn out to be the continuum version of how the JDEQSim updates its link model: In every time step, all vehicles that have just left the link are taken out of the DQ and all vehicles that have entered the link L/v time units ago (corresponding to free-flow travel time) are inserted into the DQ. Similarly, all vehicles that have just entered the link are put into the UQ and all vehicles that have left the link $L/|w|$ time units ago are only now taken out of the UQ. Further, inserting (50.6) and (50.7) into (50.4) and (50.5) yields

$$R(t) = \min\left\{\hat{\varrho}L - \text{UQ}(t), \hat{q}\delta\right\} \tag{50.10}$$

$$S(t) = \min\left\{\text{DQ}(t), \hat{q}\delta\right\}, \tag{50.11}$$

which again corresponds to how JDEQSim evaluates the boundary conditions of a link: The amount of flow allowed to enter the link is limited by the space in its UQ and the amount of flow allowed to leave the link is limited by the number of vehicles in its DQ.

A mobsim that implements the rules Equation (50.8), Equation (50.9), Equation (50.10) and Equation (50.11) implements a KWM-consistent link model. This is almost the case for the JDEQSim, which, in its implementation as of December 2014, exhibits the sole inconsistency of not limiting the link's inflow to its flow capacity. The QSim turns out to be a particular instance of the same model where backward wave speed is set to $|w| = L/\delta$. Inserting this into Equation (50.8) leads to

$$\mathrm{UQ}(t) \quad = \quad \mathrm{UQ}(t-\delta) + \delta\left[q^{\mathrm{in}}(t-\delta) - q^{\mathrm{out}}(t-\delta)\right], \tag{50.12}$$

which represents the total number of vehicles in the entire link. This corresponds to QSim behavior, where inflow to a link is limited only by the available space in the link as a whole. Letting $|w| = L/\delta$ means that the QSim behaves like a KWM with an extremely high backward wave speed, which physically means that a queue on the link does not dissolve from its downstream end but moves ”en block” over the link.

50.3 Node Model

All mobsims in MATSim implement the same node model. Surprisingly, this node model can be traced back at least to (Cetin et al., 2003, under the name of "fair intersections"), while the literature establishing its consistency with the KWM is only a few years old (Tampere et al., 2011; Flötteröd and Rohde, 2011; Corthout et al., 2012).

Nodes in MATSim have no spatial dimension; they merely connect up- and downstream links. Tampere et al. (2011) specify a set of requirements for a (continuum) node model to be consistent with the KWM. They require that the flow through the node shall be maximized subject to the following constraints:

1. Flows are non-negative and conserved within the node. This means that vehicles cannot drive backwards and they must neither disappear nor appear within the node.
2. Flow ratios comply with exogenously specified turning fractions. For instance, if it is specified that 20 % of the outflow of link i shall turn into link j, then the amount of flow that actually advances from link i into link j shall indeed be 20 % of the flow that actually leaves link i.
3. Sending flows of upstream links and receiving flows of downstream links are not exceeded. This is explained in Section 50.2.
4. The invariance principle of Lebacque and Khoshyaran (2005) is satisfied. The most important intuitive implication of this principle is that the advancement of a queuing vehicle is not affected by the vehicles behind it.
5. A "supply constraint interaction rule" is satisfied. It defines how the limited receiving flow of a downstream link is shared by competing upstream links: in practical terms, a right-of-way specification.

Flötteröd and Rohde (2011) specify an "incremental node model" that satisfies these requirements and also provide an intuitive, computationally efficient solution algorithm. In each simulation time step, this algorithm incrementally (hence the name) moves flow from upstream links into downstream links. It does so such that all the previously enumerated constraints are satisfied anytime during the transfer, terminating only once no more flow can be moved. Thus, the ultimately moved flows also comply with all constraints and are maximal.

Now consider the code documentation of MATSim's `queuesim.QueueNode.moveNode` (as of December 2014):

```
Moves vehicles from the inlinks' buffer to the outlinks where
possible. The inLinks are randomly chosen, and for each link all
vehicles in the buffer are moved to their desired outLink as long as
there is space. If the front vehicle in a buffer cannot move across
the node because there is no free space on its destination link,
the work on this inLink is finished and the next inLink's buffer is
handled.
```

This is an informal description of how the incremental node model of Flötteröd and Rohde (2011) works, given that one adopts the conventions that the sending flow of a link is stored in its "buffer" and that the receiving flow of a link is labeled here as free space in (the upstream queue of) that link. A more detailed inspection of the underlying implementation reveals no inconsistencies with incremental node model specification.

There are two aspects of the MATSim node model that are not reflected by the above code comment.

- The sending flow of an upstream link may be limited by an outflow capacity below the flow capacity Equation (50.1) of that link; for instance, to approximate a capacity reduction resulting from a downstream traffic light. This is still consistent with the framework described above.
- The selection probability of "inLinks" is proportional to their flow capacity, meaning that links with higher capacity send, on average, more flow. This is again consistent with Flötteröd and Rohde (2011) and constitutes a concrete "supply constraint interaction rule", as required by Tampere et al. (2011).

The relative simplicity of MATSim's intersection logic may be refined in many ways. For instance, turning pockets may be added and conflicts within intersections may be modeled (cf. Chapter 12). However, some caution is needed when implementing such extensions. The present node model is, due to its simplicity, guaranteed to yield unique node flows. This property needs to be revisited when implementing more complicated specifications (Corthout et al., 2012).

50.4 Summary

This chapter demonstrated that MATSim's mobility simulation is already very close to implementing a particle-discretized instance of the KWM. For full consistency, one needs to (i) use the JDEQSim (or to implement a realistic backward wave speed in the QSim) and to (ii) limit the inflow of a link by its flow capacity (which corresponds to the maximum of its triangular FD).