CHAPTER THREE

Source Calculus – The Formalist Line of Argumentation

The formalist line of argumentation

Source Theory is at an attempt to elucidate the basic concepts of epistemology by creating a formal calculus and using it to draw conclusions in this and other areas. The calculus and its use thus constitute an attempt at a logical procedure in epistemology.

The formal calculus is constructed with the accepted axiomatic structure, with concepts, axioms and theorems. The basic elements of the calculus are: **data**, **sources** and **transmission**. These were defined informally in the previous chapter. Other concepts – including major ones such as **adoption** and **system** – are defined formally, using the basic concepts.

Sources, data and transmission

Some of the basic concepts of Source Calculus were defined above in Chapter Two. Nevertheless, for the sake of clarity I will repeat some of the definitions here briefly, without the explanations and elaborations added above.

> A **datum** is an information unit.
>
> A **truth source**, or a **source** for short, is an object that supplies a datum.
>
> **Transmission** is the act of a source supplying a datum.
>
> A **database** is the set of all the data transmitted from a given source or source model.

How to cite this book chapter:
Brown, B 2017 *Thoughts and Ways of Thinking: Source Theory and Its Applications.* Pp. 11–42. London: Ubiquity Press. DOI: https://doi.org/10.5334/bbh.d. License: CC-BY 4.0

Sources and data are objects. I use the word *object* in its widest sense, i.e., as denoting a "thing" in contrast to a "state of things" or the like. In the Source Calculus data are represented in the form of sentences. These sentences are nevertheless considered objects in that they can be categorized as elements of sets, so that the laws of set theory can be applied to them; and as terms within predicates, so that the laws of predicate calculus can be applied to them (in spite of my reservations about this calculus, which I hope to discuss elsewhere). Therefore, when a sentence (datum) appears in the form of a variable we can quantify it. The quantifiers that are used here are those used in predicate calculus – that is, the existential and the universal quantifiers.

Since sources, too, are objects, this is the case for them as well. When they are discussed in the predicate calculus, they may appear as either variables or constants, and they can be bound by quantifiers.

For brevity, if a variable appears without a quantifier, this means that it is bound by the universal quantifier. Only when both the existential and the universal quantifiers appear in the same sentence will the universal quantifier be used explicitly.

The first four Greek letters, α, β, γ, δ, are used to represent the variables that denote sources, and so does μ, denoting a particular type of source which will be specified below. These are sometimes followed by a colon, which is the **transmission sign** : α:…, β:…, γ:…, δ:…

The letters a, b, c, d, h, i, m, sometimes indexed, are used to represent the constants, followed by the transmission sign, a colon a:…, b:… c:…, d:…. The first four letters denote ordinary sources, while the letters h, i and m denote particular sources, as specified below.

A few constants should be introduced. At this stage I will describe these sources informally and briefly, but most of them are defined and discussed at greater length below.

The speaking self, denoted by the Latin letter i: The basic "source" is the speaking self. The speaking self is the agent using the calculus, who transmits the rest of the sources and data to a hearer or reader. In the case of this book, the speaking self is the text of the book, or its author, but each reader may well replace it with his or her own "I". (It might be possible to develop the discussion to involve several speaking selves, but we do not need to consider this complicating possibility here.) In practice, the speaking self is not a source and does not function as one, but functions rather as the subject to which all the sources direct their messages. Therefore, when we use a source variable, it is not always possible to posit the speaking self in it, and when this is the case, I will state it explicitly.

A community, denoted by the Latin letter h: A community is an impersonal source representing a group of sources, most often people, or the vast majority of such a group. The letter h, which denotes a community *in abstracto*, is often followed by an index, to denote a particular community, or a bracketed expression, to denote that the members of the community share a common source or sources, Thus, hf can denote the community of French speakers, while h(a) denotes the community of all the sources that adopt a.

The lower-case Greek letters φ (phi), ψ (psi), ρ (rho) and σ (sigma) are used to denote sentence variables, but there are also special sentences that are denoted by τ (tau), which are defined below.

The small Latin letters p, q, r, s are used to denote sentence constants, while t is used to denote a sentence constant for sentences of the τ type.

The sentence a:p is thus to be read as "a transmits the datum p", or "the datum p is transmitted by a". A sentence of this type, i.e., a sentence reporting the fact that a datum is transmitted by a source, is called a **transmission sentence**.

Note 1: All the sources discussed in the Source Calculus are available to the speaking self. This is because in every transmission sentence (say, a:p) the speaking self is the source that transmits the very fact of the transmission (i:a:p).

Note 2: A basic assumption of the Source Calculus is that when a source transmits a sentence, it "claims" that it is true, and thus, if the source is a person, it may be assumed that he or she also "believes" the sentence.

Indeed, in the human context (e.g. when the source is a person), we can speak about claiming and belief without using quotation marks; when we are speaking about a non-human source (e.g., a perceptual sense), however, it obviously cannot claim or believe anything, in the narrow sense of these words. In such cases, what is meant is that the data transmitted by the source appear to the sources that receive them as data that are presumed to be true. For our purposes we will refer to a source as "he" or "she" if the source is clearly a person, and as "it" otherwise.

Just as there can be direct transmissions, there can also be indirect transmissions. A direct transmission is a situation in which the source transmits a "nuclear" datum, such as b:p. An indirect transmission is a situation in which the datum transmitted by the source is itself a transmission sentence, such as a:(b:p), which can actually be written in such instances as a:b:p without the brackets. In other words, indirect transmissions are situations in which one source transmits something that was transmitted to it by another source. Thus, for example, "a:b:p" means "a transmits the datum that b transmits the datum that p", and so on without restriction. In such cases we say that a's transmission of b:p is direct and b's transmission of p is direct, but a's transmission of p is indirect.

In this sort of situation, we call the source that transmits the nuclear datum (here, b) the "primary source", and the source that transmits the sentence transmitted by the primary source (here, a) the "secondary source". If there is another source that transmits the datum of the secondary source, it is called the "tertiary source", and so on. The speaking self is never counted in the ordered list of sources.

The act of transmission is not transitive. In the case under discussion, a is not necessarily claiming that p is true, nor does it necessarily "believe" p, since it is not the one who is transmitting it. Rather, what it is claiming is only that b:p is true. In contrast, b is indeed claiming that p is true. This is the case for all indirect transmission.

We also consider datasets. A **dataset** is a set all of whose members are data. The letters Φ and Ψ denote dataset variables, while the letters P and Q denote dataset constants.

$\Phi=\{\varphi,\psi,\ldots\}$
$P=\{p,q,\ldots\}$

As defined above, a database is a set of all the data transmitted by a particular source or source model. Such a set is indicated by writing the letter denoting it to the right of the letter that denotes the set: $P\alpha \equiv \text{def } \{\varphi\}|\alpha:\varphi$

For our purposes, the universal set, denoted U, is the set of all sentences transmitted by i or by i's sources.

$U=P(i, \alpha|i:\alpha, i: \alpha:\ldots)=\{\varphi\}|i:\varphi,i:\alpha:\ldots\varphi$

All the sets we discuss are subsets of this set: $\Phi,\Psi\subset U$

Now we can establish the WFF rules.

φ is a WFF if it can be formulated as a meaningful sentence.

If φ is a WFF, then $\neg\varphi$ is a WFF.

If φ is a WFF, then $\alpha:\varphi$ and $\alpha:\neg\varphi$ are WFFs.

If φ_1, φ_2, φ_3, … are WFFs, and it is given that $\Phi=\{\varphi_1, \varphi_2, \varphi_3, \ldots\}$, then Φ is WFF and therefore $\alpha:\Phi$ is also a WFF.

If Φ and Ψ are WFFs, then

$\Phi\cup\Psi$
$\Phi\cap\Psi$
$\Phi-\Psi$
$\Phi\subset\Psi$
$\Phi\not\subset\Psi$
$\varphi\in\Phi$
$\varphi\notin\Phi$

are WFFs, where the connective signs have the meaning assigned to them in set theory.

If φ and ψ are WFFs, then

$\neg\psi; \neg\rho$
$\psi\wedge\rho$
$\psi\vee\rho$
$\psi\oplus\rho$
$\psi\rightarrow\rho$
$\psi\leftrightarrow\rho, \psi\equiv\rho$
$\psi\vdash\rho$

are WFFs, where the connective signs have the meaning assigned to them in predicate calculus.

At this point we can establish a number of axioms:

Axiom 1: The source axiom

$\forall(\varphi)\exists(x)x{:}\varphi$

Axiom 2: The axiom of the speaking self

$\varphi\equiv i{:}\varphi$

Every sentence (in the text at issue) is transmitted by the speaking self (of that text).

Note 1: The axiom refers to the greater sentence, not to the nuclear sentence.

Note 2: In view of the source axiom, φ should not have been considered as UFF, as it seems to present a datum without a source. The only reason it could be recognized as UFF is thanks to the equivalence of the axiom of the speaking self, which states that the apparently sourceless form "φ" is actually an abridged formulation of "$i{:}\varphi$".

Note 3: Note: $i{:}\varphi$ is also a sentence, so the axiom of the speaking self implies that $i{:}\varphi\rightarrow i{:}i{:}\varphi$, and so on *ad infinitum*

Axiom 3: The axiom of the sources of i

$i{:}\varphi\rightarrow\exists(x)(x\neq i)\ i{:}x{:}\varphi$

Every sentence transmitted by i is transmitted to i by a source different from i.

Axiom 4: The axiom of the credibility of the source about itself (in short, the self-credibility axiom).

$\alpha{:}\alpha{:}\varphi\rightarrow\alpha{:}\varphi$

If a source "claims" that it itself is transmitting a particular datum, then it is indeed transmitting that datum (compare: Williamson 2000, Chapter 11).

Can we also establish the opposite, $\alpha{:}\varphi\rightarrow\alpha{:}\alpha{:}\varphi$? This statement means that whenever a source transmits a given datum it also "claims" that it transmits it. In order to make such a claim, it obviously has to be "aware" of the fact that it is transmitting this datum. This is not always true, so we cannot maintain that it is so for all sources. However, we can maintain it for the speaking self:

The theorem of the speaking self's claim of transmission:

$i{:}\varphi\leftrightarrow i{:}i{:}\varphi$

Proof: This follows from the axiom of the speaking self and the self-credibility axiom.

Note: This statement is also intuitively correct, since in the Source Calculus every claim made by the speaking self is a claim that appears as part of the line of argumentation, and since this argumentation is presented (to the reader) by the speaking self, the speaking self must be aware of it.

Axiom 5: The axiom of the distribution of conjunctive transmissions
$\alpha:(\varphi\wedge\psi)\equiv \alpha:\varphi\wedge\alpha:\psi$

Axiom 6: The axiom of the distribution of implicative transmissions
$\alpha:(\varphi\rightarrow\psi)\rightarrow(\alpha:\varphi\rightarrow\alpha:\psi)$

This implies that the same is true in the biconditional as well:

The theorem of the distribution of biconditional transmissions
$\alpha:(\varphi\leftrightarrow\psi)\rightarrow(\alpha:\varphi\leftrightarrow\alpha:\psi)$

This axiom is weaker than the previous one since the connective between the antecedent and the consequent is unidirectional – a material implication – in contrast to the previous one, in which the connective is bidirectional – equivalence.

The reason the connective has to be unidirectional is that if we assumed that it is bidirectional, this would mean that the source α would be subject to the rules of logic, but in Source Calculus the sources (except for the speaking self, as explained below) are not subject to these rules.

Note: The distribution of transmissions does not apply to the connective "or". Consider, for example, the sentence $a:(p\vee\neg p)$. This sentence states that a is stating a sentence that is a tautology, and so he is necessarily making a true statement. In contrast, the distributive sentence $a:p\vee a:\neg p$ says something else entirely – namely, that a may be telling the truth or he may be stating a falsehood. The same is true for the exclusive or. However, the distribution of disjunctive transmissions can take a more banal form:

$\alpha:(\varphi\vee\neg\varphi)\equiv \alpha:(\alpha:\varphi\vee\alpha:\neg\varphi)$
$\alpha:(\varphi\oplus\neg\varphi)\equiv \alpha:(\alpha:\varphi\oplus\alpha:\neg\varphi)$

When a particular source α does not transmit that φ and does not transmit that $\neg\varphi$, then it can be said to be "silent", and no transmission sentence will appear. However, sometimes a source may state affirmatively that φ is possible and $\neg\varphi$ is also possible. In such a case, the datum will be denoted with an inverted slash between the two possible data. We define this as follows:

$\alpha:(\varphi\backslash\neg\varphi)\equiv_{def} \alpha:(\alpha:\varphi\vee\alpha:\neg\varphi)$

Such a state is one of non-decision, and is called ***non liquet.***

In practice, every source can have one of three attitudes to any meaningful datum: to transmit it, not to transmit it, or to avoid making a decision about it (these can be compared to, but do not fully overlap, the classical doxastic position: belief, disbelief and suspension of judgment; Steup 1966: 7). In light of this we can establish the following axiom:

<u>Axiom 7: The axiom of non-transmission</u>
$\neg\alpha{:}\varphi\equiv\alpha{:}(\neg\varphi\vee(\varphi\backslash\neg\varphi))$

(This could be written without the internal parenthesis, but they are used for clarity).

Note: In this way, a negative transmission sentence can be turned into an affirmative one.

Adoption

The word **adoption** is used to denote a situation in which a source states that he believes data transmitted by another source, sometimes subject to certain conditions. The sentence in which the attitude of adoption is stated is called an **adoption sentence**. Adoption sentence variables are denoted by t and their constants are denoted by t.

Full adoption of a source is an act in which one source transmits the message that he accepts everything that a given source transmits as true. This act is denoted by the **adoption sign**, which is two colons between the adoptive source and the adopted source. $\alpha{::}\beta$ (read: alpha adopts beta) is therefore defined as:

$$\alpha{::}\beta\equiv\text{def }\alpha{:}(\beta{:}\varphi\rightarrow\varphi).$$

The **rejection** of a source is the opposite of adoption. We can use the rejection sign ÷÷ to denote it:

$$\alpha\div\div\beta\equiv\text{def }\alpha{:}(\beta{:}\varphi\rightarrow\neg\varphi).$$

A specific type of adoption is **exclusive adoption**, in which the adoptive source adopts one particular other source and rejects all others. This type of adoption is not used very frequently, and is marked by X::

$$\alpha X{::}\ \beta\equiv\text{def }\alpha{:}(\beta{:}\varphi\leftrightarrow\varphi)$$

A source can adopt more than one other source. This means that it accepts the data transmitted from these sources. As mentioned above, when there is more than one source, the subject often has to determine the division of labor among them, i.e., which source is responsible for which type of data, and this entire complex (the sources and the division of labor among them) is what we call the source model. A model will be denoted by a small m followed by an indexical number: m1, m2, etc. Just as a source can adopt another source, it can adopt a source model. A model requires conditional adoption, and this issue is addressed below, after the term is explicated.

Our senses can provide good examples of division of labor in the human context. Most of our senses operate automatically on different qualities. Our ears do not see colors, just as our eyes do not hear sounds. However, there are some qualities that are transmitted by two or more sources. These create a conflict, or contradiction, between the sources, which requires the conditioning of at least one of them (as discussed below).

Any adoption of two or more sources requires a source model. When we want to state the model, we will elaborate the relation between the sources, defining it as a model mn (when n denotes a number) and writing that the source adopted mn; when we can allow it to remain unspecified, we will, for brevity's sake, denote it simply by stating that the subject adopted the two sources in common: a::(b,c).

$$\alpha::(\beta,\gamma)\equiv\text{def }(\alpha:(\beta:\varphi\rightarrow\varphi)\wedge\alpha:(\gamma:\psi\rightarrow\psi))$$

This notation denotes that α adopted both β and γ, without specifying what it will transmit in cases of conflict between their data .

If so far we have seen that τ sentences are of the form $\alpha::\beta$; now we see that they can also be of the form $\alpha::(\beta1, \beta2 \ldots)$ etc.

When we wish to specify the adoption to which the τ sentence refers we will write it in brackets after the letter t. Thus, t(a::…) will mean any adoption sentence in which the adoptive source is a; t(…::a) will mean any adoption sentence in which the adopted source is a; and t(a::b) will mean the particular adoption sentence a::b.

Note: If a::(b:φ→a:φ) then a fully adopts b. But when a:(a:φ→b:φ) a only claims the full adoption of a by b, to which b itself does not necessarily subscribe.

At this point we can state another axiom.

<u>Axiom 8: The self-adoption axiom</u>

$\alpha::\alpha$

Every source adopts itself – that is, every source accepts the data it transmits as true.

This can also be formulated as follows: $\alpha:(\alpha:\varphi\rightarrow\varphi)$.

Note: The self-adoption axiom resolves the liar paradox. If we formalize the liar paradox in the language of Source Calculus, it states the premise $i{:}(i{:}\varphi \rightarrow \neg\varphi)$ and the premise $i{:}\varphi$, and then asks whether the conclusion is $i{:}\varphi$ or $i{:}\neg\varphi$. But according to the axiom of self-adoption, the first premise is necessarily false, and so the question does not arise.

Since adoption sentences are data, the distribution axiom can apply to them, as follows:

<u>The conjunctive adoption distribution theorem</u>:
$\alpha{:}((\beta{:}\varphi \rightarrow \varphi) \wedge (\gamma{:}\psi \rightarrow \psi)) \equiv (\alpha{:}(\beta{:}\varphi \rightarrow \varphi) \wedge \alpha{:}(\gamma{:}\psi \rightarrow \psi))$

If we reverse the sides, we get:

$\alpha{::}(\beta,\gamma) \equiv (\alpha{::}\beta \wedge \alpha{::}\gamma)$

<u>The implicative adoption distribution theorem</u>:
$\alpha{:}((\beta{:}\varphi \rightarrow \varphi) \rightarrow (\gamma{:}\psi \rightarrow \psi)) \rightarrow (\alpha{:}(\beta{:}\varphi \rightarrow \varphi) \rightarrow \alpha{:}(\gamma{:}\psi \rightarrow \psi))$

That is,

$\alpha{:}((\beta{:}\varphi \rightarrow \varphi) \leftrightarrow (\gamma{:}\psi \rightarrow \psi)) \rightarrow (\alpha{::}\beta \leftrightarrow \alpha{::}\gamma)$

This implies that the same is true for the biconditional.

<u>The biconditional adoption distribution theorem</u>:
$\alpha{:}((\beta{:}\varphi \rightarrow \varphi) \leftrightarrow (\gamma{:}\psi \rightarrow \psi)) \rightarrow (\alpha{:}(\beta{:}\varphi \rightarrow \varphi) \leftrightarrow \alpha{:}(\gamma{:}\psi \rightarrow \psi))$

That is,

$\alpha{:}((\beta{:}\varphi \rightarrow \varphi) \leftrightarrow (\gamma{:}\psi \rightarrow \psi)) \rightarrow (\alpha{::}\beta \leftrightarrow \alpha{::}\gamma)$

The last theorem brings us to another issue. So far, we have been discussing full adoption, but the biconditional adoption distribution theorem leads us to a discussion of partial adoption.

Partial or **conditional adoption** occurs when the adoptive source accepts the data transmitted to it by the adopted source as true if and only if a particular condition holds. Let p' be the conditional sentence. This condition will be called **the adoption restriction condition**. Such partial adoption will be denoted by a formula in which the conditional sentence, followed by a slash, is placed between the adoption symbol and the adopted source. Partial adoption is thus defined as follows:

$\alpha{::}(p'/\beta) \equiv_{def} \alpha{:}(p' \leftrightarrow (\beta{:}\varphi \rightarrow \varphi))$

Even an exclusive adoption can be conditional:

$$\alpha X{::}(p'/\beta) \equiv_{def} \alpha{:}(p' \leftrightarrow (\beta{:}\varphi \leftrightarrow \varphi))$$

The definition of partial adoption leads to:

<u>The theorem of the distribution of partial adoption</u>:
$\alpha{::}(p'/\beta) \rightarrow (\alpha{:}p' \leftrightarrow \alpha{:}(\beta{:}\varphi \rightarrow \varphi))$

Proof:

$\alpha{:}(p' \leftrightarrow (\beta{:}\varphi \rightarrow \varphi)) \equiv (\alpha{:}p' \leftrightarrow \alpha{:}(\beta{:}\varphi \rightarrow \varphi))$ by the theorem of the distribution of biconditional transmissions
$\alpha{:}(p' \leftrightarrow (\beta{:}\varphi \rightarrow \varphi)) \equiv \alpha{::}(p'/\beta)$ by the definition of partial adoption
$\therefore\ \alpha{::}(p'/\beta) \rightarrow (\alpha{:}p' \leftrightarrow \alpha{:}(\beta{:}\varphi \rightarrow \varphi))$

QED
The conditioning can also apply to two or more sources. Moreover, it may be different for each of the sources:

$$\alpha{::}(p'/\beta, q'/\gamma) \equiv_{def} \alpha{:}((p' \leftrightarrow (\beta{:}\varphi \rightarrow \varphi)) \wedge (q' \leftrightarrow (\gamma{:}\psi \rightarrow \psi))$$

Note: The distribution of biconditional adoption, as presented above, is an example of partial adoption, according to the definition presented here. In such a situation the adoption of α and β are conditioned on each other.

When the sentence of the condition of restricted adoption p' (or q') is a tautology, the adoption becomes full. This shows that partial adoption includes the possibility of complete adoption, although the reverse is not the case. This implies:

$$\alpha{::}\beta \rightarrow \alpha{::}(p'/\beta)$$

In the following discussions we will mostly make use of partial adoption, which has the broadest range of application. In many of the discussions, the concrete content of p' is unimportant. For these cases we will use an abbreviated symbol of partial adoption: /: . We can define this symbol as follows:

$$\alpha/{:}\beta \equiv_{def} \alpha{::}(p'/\beta)$$

A conditional exclusive adoption in which the condition is not specified will be denoted by X/:, as following:

$$\alpha X/{:}\beta \equiv_{def} \alpha X{::}(p'/\beta)$$

Note: The difference is that in the formula $\alpha/{:}\beta$ the condition is not specified. Thus we will use it only in cases where the identity of the condition is not relevant to the issue under discussion, i.e. in cases where only the conditional nature of the adoption is at stake.

In a different formulation we can therefore state:

<u>The adoption relation theorem</u>
$\alpha{::}\beta\rightarrow\alpha/{:}\beta$

This is also true, *mutatis mutandis*, for the adoption of more than one source.

If, on the other hand, an adoption restriction sentence is a contradiction, then the adoption does not hold. This situation constitutes the rejection of the source under consideration.

Sometimes there is a situation in which the adoption restriction condition establishes that the data transmitted by the adopted source belong to some dataset P. This is called **(ordinary) compartmentalization**. In such a situation the adoption restriction condition is denoted by placing the membership sign, followed by name of the set, before the slash (this is a convenient denotation, even though it is not elegant):

$$\alpha{::}(\in P/\beta)\equiv\text{def }\alpha{:}((\varphi\in P)\leftrightarrow(\beta{:}\varphi\rightarrow\varphi))$$

To be sure, the opposite situation, in which *not* belonging to the dataset is the condition, is also possible. In that case the situation will be notated by the non-membership sign:

$$\alpha{::}(\notin P/\gamma)\equiv\text{def }\alpha{:}(\varphi\notin P\leftrightarrow(\gamma{:}\varphi\rightarrow\varphi))$$

Or, if we use U to denote the Universal Set:

$$\alpha{::}(\notin P/\gamma)\equiv\text{def }\alpha{:}(\varphi\in(U\text{-}P)\leftrightarrow(\gamma{:}\varphi\rightarrow\varphi))$$

The main benefit of compartmentalization is obtained when it is used for more than one source:

$$\alpha{::}(\in P/\beta,\in Q/\gamma|(Q\cap P=\varnothing))\equiv\alpha{:}((\varphi\in P\leftrightarrow(\beta{:}\varphi\rightarrow\varphi))\wedge(\varphi\in Q\leftrightarrow(\gamma{:}\psi\rightarrow\psi))|(Q\cap P=\varnothing))$$

Compartmentalization is therefore an excellent example of the division of labor among sources.

Complementary compartmentalization occurs when the two (or more) adopted sources are "authorized" for complementary sets:

$$\alpha{::}(\in P/\beta,\notin P/\gamma)\equiv\alpha{:}((\varphi\in P\leftrightarrow(\beta{:}\varphi\rightarrow\varphi))\wedge(\varphi\in(U\text{-}P)\leftrightarrow(\gamma{:}\psi\rightarrow\psi)))$$

Another type of compartmentalization is a **hierarchy**. This is a situation in which a source γ is adopted on the condition that its data do not contradict those of another source β, which has also been adopted. In such a case we will say that beta is a **superior source** in the hierarchy and γ is a **subordinate source**. Such a situation is denoted by having the superior source appear before the slash, and the subordinate source after it. In terms of compartmentalization this means that the adoption of γ is compartmentalized to data that do not contradict β's data.

If the dataset transmitted by β is denoted $P\beta$ (as above), then the hierarchy is defined as follows:

$$\alpha{:}(\beta/\gamma)\equiv\mathrm{def}\ (\alpha{::}\beta\wedge\alpha{::}(\varphi\notin P\beta\leftrightarrow(\gamma{:}\varphi\rightarrow\varphi)))$$

It can also be defined somewhat more simply:

$$\alpha{::}(\beta/\gamma)\equiv\mathrm{def}\ \alpha{:}((\beta{:}\psi\rightarrow\psi)\wedge\neg(\gamma{:}\neg\psi)\leftrightarrow(\gamma{:}\varphi\leftrightarrow\varphi))$$

So far I have presented possible interrelations between sources in the form of adoption by another source, α. This serves as a unifying factor, which determines the order of the sources it adopted. However, we can describe this order abstractly and independently as a unit ready for adoption as a whole. This presentation, which allows great brevity, is in the form of a model. As mentioned above, a model is presented by a small m, usually with a numeral index, or, when speaking about a variable, by the Greek μ. For example, if we want to introduce a model of complementary compartmentalization, as mentioned, we may describe it as a model called m1:

$$m1 = (\in P/\beta, \notin P/\gamma).$$

For short, we can simply say that m1 itself is a theoretical source, and write:

$$m1{:}\ \forall\varphi\ (((\varphi\in P)\rightarrow\varphi)\wedge((\varphi\notin P)\rightarrow\gamma))$$

If α applies this model, we can simply state that it adopted m1:

$$\alpha{::}m1 \equiv \alpha{::}(\in P/\beta, \notin P/\gamma)$$

And if α adopts m1 exclusively we write: $\alpha X{::}m1$

This notation saves us the need to elaborate complex source relations whenever we mention them. In terms of content, we will treat the adoption of a model as an adoption of sources.

Let us continue the discussion of our senses. As I wrote above, most of our senses are compartmentalized. Since they transmit different qualities, they do not have occasion to conflict with one another: Our ears do not see and our eyes do not taste. There are, however, some qualities that are transmitted by two or more sources . These create a conflict between the sources that requires the conditioning of at least one of them – or the creation of a hierarchy. Consider the following examples:

1. The sense of sight transmits that the paint on the banister is dry; the sense of touch transmits that it's wet.
2. The sense of sight transmits that the paint on the banister is wet; the sense of touch transmits that it's dry.

We can imagine at least five consistent responses to these situations:

a) Believing the data transmitted by the sense of sight in both cases.
b) Believing the data transmitted by the sense of touch in both cases.
c) Believing the more desirable datum (dry paint) in both cases.
d) Believing the less desirable datum (wet paint) in both cases.
e) *Non Liquet*

Options a and b give priority to the chosen datum according to the superior status of its source; options c and d give priority according to content, probably in relation to the agent's predispositions (cautious or nonchalant).

One particular type of compartmentalization is called **external decision.** In this situation, it is established that whenever source α encounters a contradiction between β's data and γ's data, then a fourth source, δ, determines which datum source α will believe. In this sort of situation, δ is called the **deciding source**, and the situation is denoted by two slashes between δ, on the one hand, and β and γ, on the other:

$$\alpha::(\delta//(\beta,\gamma))\equiv\alpha:(((\delta:\varphi\leftrightarrow\alpha/:\beta:\varphi))\wedge(\delta:\neg\varphi\leftrightarrow(\alpha/:\gamma:\neg\varphi))|\delta\neq\alpha,\beta)$$

Note 1: External decision should not be considered a case of hierarchy, in which β and γ are subordinate to δ, since δ's supremacy comes into play only in case of a contradiction between the data of β and γ, while in other cases they may well be superior to δ.

Note 2: There can also be situations in which δ's transmission of a datum is conditioned in various ways.

Note 3: When an adoption restriction sentence states that a datum belongs to a certain set, and this set is empty, the adoption is defeated, and so this situation is one of rejection of the source at issue, as defined above.

Note 4: The concept of a hierarchy helps us explicate the concept of defeasibility more precisely. Defeasibility, which has been proven to be a fruitful concept in contemporary logical and philosophical discussions, is a state in which a datum from a subordinate source is transmitted at first without the transmission of a contradictory datum from a superior source, and so it is worth believing, yet later on a contradictory datum from the superior source is transmitted, which, according to the hierarchy, defeats the previous datum.

All the forms presented so far – ordinary conditions, compartmentalization, hierarchies, external decision and rejection – are specific forms of partial adoption which were developed by substituting certain phrases in the defining formula for partial adoption.

Now we need to distinguish between direct and indirect adoption.

Direct adoption occurs when one source adopts another without the intervention of a third source. For example, a::b represents direct adoption.

Indirect adoption occurs when one source adopts a second one, and the second source adopts a third one. For example, a::b::c represents a situation in which a adopts b directly and b adopts c directly, but a adopts c indirectly. In such a situation we say that a adopts c **by virtue of** b.

Adoption sentences too can be combined with transmission sentences – that is, one source can adopt another source, which transmits a certain datum. This situation is called **transmission by virtue of adoption**. When the adoption is complete, the situation is denoted as a::b:p. This sentences reads, "a transmits that p by virtue of having fully adopted b". Of course, in such a case a also accepts p to be true by virtue of that adoption. If the source is a person, we would say he **believes** in p by virtue of the adoption of b. As with transmission, we call the source that transmits the nuclear datum (here b) the **primary** source, and the source that transmits the primary source's transmission sentence (here a) the **secondary** source. If another source transmits the secondary source's transmission, it is called the **tertiary** source, etc.. Here too the speaking self is never counted in the ordered list of sources.

A source's belief in a datum that it transmitted is called **direct belief,** while belief in virtue of another source is called **indirect belief.**

Now **transmission in virtue of full adoption** will be defined as follows:

$$\alpha::\beta:\varphi \equiv \text{def } (\alpha::\beta \wedge \beta:\varphi)$$

But according to the definition of full adoption, using modus ponens, we deduce that $\alpha:\varphi$. We can state this as a theorem:

<u>The indirect adoption theorem</u>

$$\alpha::\beta:\varphi \rightarrow \alpha:\varphi$$

Proof: By virtue of the definition of full adoption and modus ponens.

This means that, in contrast to transmission, full adoption is transitive.

Note: This implies that direct belief does not have any logical priority over indirect belief.

These situations must be distinguished from that of **mediated adoption.** When a adopts c, but receives c's data from another source, b, we say that a adopts c by virtue of b's mediation. This situation is denoted a::(b:c) and is defined as follows:

α::(β:γ)≡def α:(β:γ:$\varphi\rightarrow\varphi$)

In mediated adoption the adoptive source adopts the mediated source not as a source of data about the world, but as a source of data about other data being transmitted from another source. Later we discuss the logical character of this sort of adoption.

Note: Mediated adoption is also a sort of compartmentalization, since the adoptive source accepts the data of the mediating source as true if and only if they belong to a particular set, which is the set of transmission sentences of another source. (To be sure, the adoptive source can also adopt the mediating source for other matters as well, even in direct adoptions, but these additional adoptions are irrelevant for the mediated adoption presently under discussion.)

<u>The self-adoption theorem</u>
α::α:φ≡α:φ

Proof: By virtue of the self-adoption axiom and the self-credibility axiom.

Note: This means that whenever the expression α::α:φ appears it can be abbreviated to α:φ.

The situation of **transmission in virtue of partial adoption** is denoted α::(p'/β):φ

This means that α transmits sentence φ in virtue of partial adoption, and that p', which is the condition for adopting β, is satisfied. In the abridged form, α:/β:φ, we say that α transmits sentence φ in virtue of partial adoption, and that the unspecified condition for adopting β is satisfied. Here, too, it is obvious that α also accepts φ to be true, or, in a human context, believes it. This situation is defined as follows:

α::(p'/β):φ≡def (α::(p'/β)$\wedge$p$\wedge\beta$:φ)

The conclusion to be drawn from it is that α:φ. The sentence a::(p'/b):p therefore means, "a transmits that p and accepts p as true in virtue of the fact that it has partially adopted b, subject to the condition p'".

There can also be situations such as a:b::c:p (a claims that b adopts c, who claims that p), and so on.

Databases and systems

Database and system have already been defined, but let us recall them:

A **database** is the set of all the data transmitted from a given source or source model.

A **system** is a source or a source model together with the database that was transmitted by it.

The primary sources that transmit the data of the database or the system (and thus serve as their final justification, as explained below) will be called its **basic** sources, and we will say that the database (or the system) is **based** or **founded** on them.

A database is denoted by the letter D, followed by the source on which it is based (in parentheses). If there is one source, the database is denoted D(a); if there are two, D(a,b); and so on. If it is based on a source model m1, we denote it as D(m1).

A system is denoted by the letter S, followed by the source on which it is based (in parentheses). If there is one source, the system is denoted S(a); if there are two, S(a,b); and so on. If it is based on a source model m1, we denote it as S(m1).

The definition of a database is thus similar to the definition of a set of source data:

$$D(\alpha)\equiv\text{def } P\alpha\equiv\{\varphi\}|\alpha{:}\varphi,\ \beta{::}\alpha{:}\varphi,\ \gamma{::}\beta{::}\alpha{:}\varphi,\ \ldots{::}\alpha{:}\varphi$$
$$D(\alpha,\beta)\equiv\text{def } P(\alpha,\beta)\equiv\{\varphi\}\cup\{\psi\}|\alpha{:}\varphi,\beta{:}\psi,\ \ldots{::}\alpha{:}\varphi,\ \ldots{::}\beta{:}\psi$$

Note: The definition of a database as a set should not mislead us. In its pure set-theoretical meaning, a set is an a-temporal entity, and is determined by its members. Consequently, if in t1 the basic sources transmit p1 and p2, these data will constitute one set, i.e. one database, and if in t2 they transmit a third datum p3, they will constitute another database, etc. However, in Source Theory a database is determined not by its data but rather by its sources, and the sources' transmissions take place in time. To bridge this gap, we define a database as the union set of all the data transmitted by the basic sources at all times.

Let's concentrate for the moment on a database based on one source, α. One of the data transmitted by α is the sentence α::α (by the self-adoption axiom). According to the notation presented above, this sentence is denoted as t(α::α) (t standing here for the adoption sentence). If so, necessarily

$$t(\alpha{::}\alpha)\in P\alpha$$

But if so, then necessarily

$$\beta{:}P\alpha\rightarrow\beta{:}(\alpha{::}\alpha)$$

Of course, this still does not mean that β adopted α; it only reported α's self-adoption. Neither does the fact that β transmits all of Pα entail that it adopted α, since the same set of data could have been transmitted by another source or by multiple sources.

The basic concepts of set theory apply to the relations between databases and data, as well as between databases and databases. Thus, a database α is said to include a database β if all the members of β are also members of α:

$$D(\alpha)\subset D(\beta) \equiv\text{def } \forall\alpha,\forall\beta\ (\alpha\in D(\beta)\rightarrow\alpha\in D(\beta))$$

In such a case we will say that database D(β) is a **subdatabase** of D(α) and that System S(β) is a **subsystem** of S(α).

Note: The sources of the subsystem may include only some of the sources of the larger system (see the definition of database above).

Another form of subsystem exists when the sources of one system fully adopt the sources of another. In that case we will say that the former is a subsystem of the latter.

<u>The theorem of the adoption-inclusion relation</u>

$$\alpha::\beta\rightarrow D(\alpha)\supseteq D(\beta)$$

Proof: The adoption of a source entails the transmission of the (data in the) database based on it. If D(α) denotes the set of data transmitted by α, then it also includes the data transmitted by α as result of the adoption of β.

This is not a biconditional, since the inclusion of the data of D(β) in D(α) may be a result of coincidence, and not necessarily a result of adoption. Consequently, there may be a case in which source α claims that all the data transmitted by source β are also transmitted by itself (namely, by source α), without the adoption of source β. In such a case α's claim will be taken as an ordinary transmission sentence and not an adoption sentence. In view of this, we will say that D(β) is an **alleged subdatabase** of S(α), and that S(β) is an **alleged subsystem** of S(α).

If α adopted β through compartmentalization, we will say that D(β) is a **compartmentalized subdatabase** of D(α) and that S(β) is a **compartmentalized subsystem** of S(α).The same is true for all the various types of compartmentalization.

Subordination to logic

In the Source Calculus, the speaking self is subject to the rules of logic (I have used this assumption throughout this chapter, and I formulate it as an axiom below). We cannot define "Logic" with a capital L here, especially nowadays, when a large number of different logics have been proposed. Clearly a "logic" is the most basic collection of the rules of rational thinking, and any attempt to define it in a principled way will bog us down in a comprehensive discussion of the nature of such thinking, which is inappropriate for the present stage of our discussion. However, we actually do not require a principled definition for our present purposes, but need only explain what sense of the term "logic" we are using here. Accordingly, the "rules of logic" I am referring to are the collection of basic inference rules of the classical propositional calculus. I will present this collection of rules here as a sort of dataset, which I call P(L). P(L)={L1…L22}, i.e. D(L) includes the set of sentences L1–L22.

Even though this set of sentences is the collection of basic rules of inference accepted in the propositional calculus, we will restate them for the completeness and clarity of our discussion:

L1: The law of identity
$\varphi\leftrightarrow\varphi$

L2: The law of non-contradiction
$\neg(\varphi\wedge\neg\varphi)$

L3: The law of the excluded middle
$\varphi\vee\neg\varphi$

L4: Modus ponens
$\varphi\rightarrow\psi$
φ
$\therefore\psi$

L5: Modus tollens
$\varphi\rightarrow\psi$
$\neg\psi$
$\therefore\neg\varphi$

L6: Hypothetical syllogism
$\varphi\rightarrow\psi$
$\psi\rightarrow\rho$
$\therefore\varphi\rightarrow\rho$

L7: Disjunctive syllogism
$\varphi\vee\psi$
$\neg\varphi$
$\therefore\psi$

L8: Constructive dilemma
$(\varphi\rightarrow\psi)\wedge(\rho\rightarrow\sigma)$
$\varphi\vee\rho$
$\therefore\psi\vee\sigma$

L9: Absorption
$\varphi\rightarrow\psi$
$\therefore\varphi\rightarrow(\varphi\wedge\psi)$

L10: Simplification
$\varphi\wedge\psi$
$\therefore\varphi$

L11: Conjunction
φ
ψ
$\therefore\varphi\wedge\psi$

L12: Addition
φ
$\therefore\varphi\vee\psi$

L13: De Morgan's theorems
$\neg(\varphi\wedge\psi)\equiv(\neg\varphi\vee\neg\psi)$
$\neg(\varphi\vee\psi)\equiv(\neg\varphi\wedge\neg\psi)$

L14: Commutativity
$(\varphi\vee\psi)\equiv(\psi\vee\psi)$
$(\varphi\wedge\psi\psi\equiv(\psi\wedge\varphi)$

L15: Associativity
$(\varphi\vee(\psi\vee\rho))\equiv((\varphi\vee\psi)\vee\rho)$
$(\varphi\wedge(\psi\wedge\rho))\equiv((\varphi\wedge\psi)\wedge\rho)$

L16: Distribution
$(\varphi\wedge(\psi\vee\rho))\equiv((\varphi\wedge\psi)\vee(\varphi\wedge\psi))$
$(\varphi\vee(\psi\wedge\rho))\equiv((\varphi\vee\psi)\wedge(\varphi\vee\rho))$

L17: Double negation
$\varphi\equiv\neg\neg\varphi$

L18: Transposition
$(\varphi\rightarrow\psi)\equiv(\neg\psi\rightarrow\neg\varphi)$

L19: Material implication
$(\varphi\to\psi)\equiv(\neg\varphi\vee\psi)$

L20: Material equivalence
$(\varphi\equiv\psi)\equiv((\varphi\to\psi)\wedge(\psi\to\varphi))$
$(\varphi\equiv\psi)\equiv((\varphi\wedge\psi)\vee(\neg\varphi\wedge\neg\psi))$

L21: Exportation
$[(\varphi\wedge\psi\to\rho]\equiv[\varphi\to(\psi\to\rho)]$

L22: Tautology
$\varphi\equiv\varphi\vee\varphi$
$\varphi\equiv\varphi\wedge\varphi$

Now we can establish the axiom regarding the acceptance of these rules by the speaking self.

Axiom 9: The logicality of the speaking self
This axiom consists of three sentences:
(1) $i{:}P(L)$
(2) $\forall\alpha\ \neg\ i{::}\alpha$
(3) $\forall\alpha\ (i/{:}\alpha \leftrightarrow i{::}(L/\alpha))$

Note 1: This axiom is actually quite problematic, since it is based on the assumption that the speaking self is subordinate to the rules of rational thought, while we can easily imagine a situation in which the speaking self refuses to subordinate himself to them. Nevertheless, we have no choice but to make this assumption, since the basis of our discussion here is the Source Calculus, which is a logical one, and the speaking self is the agent who employs this calculus. The speaking self of a logical calculus must be subordinate to the rules of the logical subsystem. At any rate, as I mentioned in the introduction, the entire Source Calculus, being logical and rational, is like Schopenhauer's ladder, which the climber can throw away once he has reached his goal, but as long as he is using it, he has to use it as it is – that is, as part of logic.

Note 2: The fact that the speaking self is subject to the laws of logic does not mean that the sources whose data he transmits are subject to these laws. If we take the law of contradiction as an example, the speaking self can transmit information about sources that transmit contradictory data, and the like. As long as he himself does not transmit these contradictory data – i.e. claim their truth – there is no problem.

This is analogous to a situation in which a judge writes in an opinion that the witness Tom contradicted himself in his testimony, or that the witnesses Dick and Harry presented contradictory evidence. The judge who presents this situation is not contradicting himself when he reports the contradiction, unless he writes that he believes both versions of Tom's testimony, or that he believes both Dick's and Harry's versions.

Note 3: The fact that the speaking self is subject to the laws of logic makes it impossible for him to completely adopt a source whose data contradict these laws, but it does permit him to adopt it partially, as long as the adoption condition states that those data that contradict the laws of logic cannot be accepted.

Note 4: On the basis of the axiom on the logicality of the speaking self, we can leave out all mention of the speaking self's acceptance of L as a condition for any adoption. Since L is accepted inherently as a set of data prior to any adoption, whenever we mention that a source has been adopted by i, we will read into the text that this adoption is subject to L.

Many theorems can be deduced from this axiom. One example is the following:

<u>The theorem of the consistency of the speaking self's beliefs (for short, the consistency theorem)</u>

$i{:}\varphi \rightarrow \neg i{:}\neg\varphi$ (by the logical principle of the speaking self and the law of non-contradiction).

Here another theorem seems appropriate:

<u>The theorem of the hypothetical syllogism of adoption</u>

$\alpha{::}(p'/\beta)$
$\beta{::}(p'/\gamma)$
$\therefore \alpha{::}(p'/\gamma)$

Proof: By the definition of adoption, the axiom of the logicality of the speaking self, modus ponens (L4) and the hypothetical syllogism (L6). This theorem also applies to the speaking self.

I will now elaborate on the issue of avoiding contradictions between different data sources, but the discussion will only be an example, and the law of non-contradiction will serve as a model for all the laws of logic.

Contradictory data

How can the speaking self deal with contradictions among the data that have been transmitted to him? First of all, according to the axiom of the logicality of the speaking self, he clearly cannot believe both of them at the same time. But what other alternatives does he have?

Here it is important to distinguish contradictions between data from a single source from those between data from different sources. (Throughout this chapter, it is given that all the sources are different from i, that is, $\alpha,\beta,\gamma,\delta \neq i$.)

Case 1: A contradiction between data from a single source:

$\alpha{:}\varphi \wedge \alpha{:}\neg\varphi$

This means

$\alpha{:}(\varphi \wedge \neg\varphi)$ by the axiom of the distribution of conjunctive transmissions

Since it is impossible to have $i{:}(\varphi \wedge \neg\varphi)$, from i's point of view there are thus three alternatives:

1) $i{:}\varphi$
2) $i{:}\neg\varphi$
3) $\neg i{:}\varphi \wedge \neg i{:}\neg\varphi$

Alternatives 1 and 2 can be achieved only by the partial adoption of α (by the adoption axiom, as stated below, ruling out the possibility of full adoption), while alternative 3 means rejecting α, thus creating a state of *non liquet*.

If the speaking self wants to decide on one of the alternatives, he must establish a condition to rule out one of them:

$\alpha{:}(\varphi \wedge \neg\varphi) \rightarrow i{::}(p'/\alpha)$

where p' leads to the negation of φ or the negation of $\neg\varphi$.

Case 2: A contradiction between the data of two or more sources.

Here I will discuss the case of two sources, but the discussion also applies, *mutatis mutandis*, to more than two.

$\alpha{:}\varphi \wedge \beta{:}\neg\varphi$

Since it cannot be the case that $i{:}(\varphi \wedge \neg\varphi)$, from this point of view there are three alternatives:

1) $i{:}\varphi$
2) $i{:}\neg\varphi$
3) $\neg i{:}\varphi \wedge \neg i{:}\neg\varphi$

Alternatives 1 and 2 can be achieved only by the partial adoption of α and β (by the adoption axiom, as stated below, ruling out the possibility of full adoption), while alternative 3 means rejecting both α and β, at least for the present purposes, thus creating a state of *non liquet*.

The partial adoptions can occur in all the forms detailed above: ordinary adoption conditions for one of the sources, compartmentalization, hierarchy, external decision or rejection. In the present context of a contradiction between the data of different sources, these situations are activated as **consistency mechanisms**, which is what we shall call them.

In light of all this, the possible mechanisms are:

1 Adoption conditions of the regular, basic sort:

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}(p'/\alpha,\beta) | p' \vdash \neg\alpha{:}\varphi$$

or

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}(\alpha,q'/\beta) | q' \vdash \neg(\beta{:}\neg\varphi)$$

or

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}(p'/\alpha,q'/\beta) | (p' \vdash \neg\alpha{:}\varphi \vee q' \vdash \neg\beta{:}\neg\varphi)$$

2 Compartmentalization:

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}(\in P/\alpha, \in Q/\beta | (Q \cap P = \emptyset))$$

This also includes the possibility of complementary compartmentalization:

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}(\in P/\alpha, \notin P/\beta)$$

3 Hierarchy:

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}(\alpha/\beta)$$

or

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}(\beta/\alpha)$$

4 External decision:

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}(\delta//(\alpha/\beta) | \delta \neq \alpha,\beta$$

5 Rejection:

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}\alpha$$

or

$$(\alpha{:}\varphi \wedge \beta{:}\neg\varphi) \rightarrow i{::}\beta$$

Note: As mentioned, a *non liquet* situation is also one of rejection, but it involves the rejection of both of the sources α and β.

Each of these mechanisms can bring about the desired result, namely, either $i:\varphi$ or $i:\neg\varphi$.

It is important to remember that the all the consistency mechanisms are particular forms of adoption conditions, and while we could just as well have established only general conditions, it is more convenient to have all the alternatives spelled out.

Moreover, the list of consistency mechanisms is not a closed one, and there can in principle be other mechanisms, but these too are derivable from the basic form of the adoption condition.

Therefore we can establish the following theorem:

The decision theorem

In a case when two different sources transmit contradictory data, then if and only if the speaking self transmits something about these data, he can transmit only one of the contradictory data, through the (partial or full) adoption of the source that has transmitted it.

$$(\alpha:\varphi\wedge\beta:\neg\varphi)\rightarrow(\neg i:(\varphi\backslash\neg\varphi))\leftrightarrow(i::(p'/\alpha,q'/\beta):\varphi\vee i::(p'/\alpha,q'/\beta):\neg\varphi)$$

This is the case when we are given that:

1. The condition sentences p' and q' are adoption restriction sentences of the sort described above;
2. p' and q' can also be tautologies, thus rendering the partial adoption an almost full one (subject only to the rules of logic L), for either of the sources α and β (but not both, due to the theorem on the consistency of the speaking self).
3. p' and q' can also be contradictions, thus leading to the rejection of source α or β (but not both, due to the antecedent that rules out *non liquet* here).

Proof: By the definition of the *non liquet* situation and the argumentation in this section.

Justification

As stated above (Chapter Two), for any source under discussion, writing the adoption sentence together with the transmission sentence not only states the facts of adoption and transmission, but is also the **justification** for the transmitted datum. (In mediated adoption, the mediated source is seen as the source of the justification, rather than the mediating source, which is nothing but a tool for transmitting the data of the mediated source.) For any claim one can ask the transmitting source, "Why should I believe it?" The answer to this question is the justification of the claim. The justification can be either (1) another datum, which supports the one under consideration, or (2) indicating the source of the datum that the speaker had adopted. If the justification is of type (1), then this answer is itself also a claim, and so one can ask the same question about this answer as well, and so on; if the answer is of type (2), we may ask the speaker why he or she adopted that source; and then the answer will be data which support the credibility of that source; but these data will have to face the same question, and so on. The next-to-last "Why?" question is always answered "Because this and that source transmitted it to me", and if this leads to the question, "Why should I believe it?", then this is the last question in the chain, and it must necessarily be answered, "Because I have adopted this source as a source". Since this is the last "Why?" question, it is the **final justification** of the claim. (To repeat, senses, reason and the like are also sources, and therefore justifying data by using them as a final justification is no different from doing so for any other source, from the point of view of Source Calculus.) In any series of adoptions and transmissions, the rightmost adoption (i.e. the closest to the "nuclear" datum) is the final justification, and all the rest are dependent on it.

At this point one may properly ask: Why is the adoption sentence the final justification? Why not data? The choice of the sources also requires justification, and that justification, for its part, is also a datum! However, as we have clearly seen, the corroborating datum too is transmitted by a source, and that source is the justification for the belief in that datum, and so on in an infinite regress (see Chisholm 1964; Armsrtrong 1973; BonJour 1978; Lehrer 1990). However, we must insist that the ultimate point in this line is a source, not a datum. The reason for this is simple: Every datum depends on its source for its very existence, while not every source depends on a datum (even if its status as a source depends on its ability to transmit data).

This description is in the spirit of the foundationalist theory. Another possibility is granting the system justification through a coherentist theory, but this option will be ruled out below. Within the foundationalist framework, then, there are four main approaches to coping with the question of justification (see Chisholm 1964: 264; compare BonJour 1978; Lehrer 1990; Steup 1996).

1) Dismissing the question as based on false assumptions.
2) An infinite line of justification: p1 is justified by p2, which is justified by p3, and so on indefinitely.
3) A circular line of justification: p1 is justified by p2, which is justified by p1.
4) A foundationalist line of justification: p1 is justified by p2, which is accepted as "foundational" because it is one of the following: (4.1) beyond the requirement of justification; (4.2) unjustified; (4.3) self-justified; or (4.4) neither justified nor unjustified.

This list refers to any type of justification, but we need to apply it to the justification of data and adoptions. Here we encounter the infinite regress problem, but in a version peculiar of the source-datum relation. If source a transmits datum p1 and we ask him why he accepts it as true, he would answer that it is because source b transmitted it to him, and he has adopted source b. The adoption of b by a will be denoted as t1. So now we can ask what justifies t1 (namely, the adoption of b by a). Source a would answer that such-and-such data, called in common p2, justify b's credibility; if we, in turn, now ask what makes him accept p2 as true, a can answer, in the spirit of the list above, one of the following:

1) The question is based on false assumptions.
2) p2 is justified by source c, whose adoption is expressed in sentence t2; t2 is justified by data p3; which were transmitted by source d; etc. in an indefinite line.
3) p1 is justified by t1, which is justified by p1.
4) p1 is justified by t1, which is justified by p2; p2, in turn, is accepted as "foundational" because is one of the following: (4.1) beyond the requirement of justification; (4.2) unjustified; (4.3) self-justified; or (4.4) neither justified nor unjustified.
5) p1 is justified by t1, which is accepted as "foundational" because it is one of the following: (4.1) beyond the requirement of justification; (4.2) unjustified; (4.3) self-justified; or (4.4) neither justified nor unjustified.

However, some of these options should be ruled out immediately:

1) There is no evidence that the question is based on false assumptions.
2) There cannot be an indefinite line of justification of sources and data, since the number of sources is finite.
3) Circularity, or self-justification, is an invalid justification (as for coherence theories, see below).
4) Besides the essential difficulties in the four exemptions from ordinary justification, and besides the difficulty of finding data that would qualify for such a foundational character, we cannot accept data as final justifications, as proven above.

We remain, therefore, with option (5). We should now explore which of the alternative options within (5) may serve as an acceptable justification for the final t sentence.

The meaning of final justification is that the adopted source justifies itself. It does not refer to any other source, but "asserts" that it itself is the source by virtue of which the adoption is justified. This is thus a situation of self-justification. A similar situation occurs when source a justifies itself by referring to source b, which it has adopted, while source b in turn justifies itself by referring to source a, which *it* has adopted (this can also be described for a larger number of sources). This is therefore a situation of circular justification, which is only a more complex form of self-justification (compare Stich 1988). These two sorts of justification can be called **internal justification**. In contrast, the ordinary sort of justification, in which a source refers to another source that it has adopted, without the latter referring back to the former, is called **external justification.**

Internal justification can be accepted, if at all, only as part of a coherentist theory, and therefore should be ruled out in a discussion based on foundationalist premises. Thus we can dispense of option (4.3) above. Since option (4.1) is beyond the requirement of justification, (4.2) is unjustified, and (4.4) is neither justified nor unjustified, they themselves require justification. Since in normal cases justification is required, if someone thinks these can be exempted from justification – that exemption itself requires justification; the justification for the exemption will be given by certain adopted source of sources, and this adoption, on its part, also requires justification, and so we return to the infinite regress problem.

We thus remain with the coherentist alternative of self-justification as the only solution to the infinite regress problem, and therefore self-justification (or internal justification) seems to be the only possible means of final justification of a source. For the time being, we will remain with this conclusion, but we will re-examine it later.

In a situation of indirect adoption, the indirect source is externally justified, while the direct source is internally justified; in a situation of direct adoption, the source is always internally justified.

Note: From the standpoint of the Source Calculus, all truth is relative to a source. The truth value of a datum is determined by its being adopted by the speaking self. To illustrate this, let us suppose that there is an unknown source – let us call it g – that transmits absolutely true data about the world, and only such data. Consequently, the definition of truth in the terms of Source Calculus would be: p is true if g:p. However, such a source would be ideal and thus unavailable to us. Hence, from the standpoint of the Source Calculus we could write source g between any source and any data transmitted by it (for example, instead of writing a:p we could write that a:g:p, and so on). This addition, however, would not be helpful and would only complicate the notation, since g itself is unavailable to us. As a result, for our purposes truth is determined not by accord between the data and the "external world", but rather between the data presented at the moment to the agent and the data of the adopted sources of that agent. In other words, as long as we do not have access to the utopian source g, the only "external world" that can make sense to us is the sum total of the data transmitted to us by the sources we have adopted. Indeed, in this sense Source Calculus entails en internalist conception of truth.

When two sources adopt another source as their final justification, whether directly or indirectly, we say that they have a common final justification, or that they are **co-justified** sources. Thus, for example, if a::b::d and c::d, then a, b, c, and d are all co-justified (given that d itself also adopts d, by the self-adoption axiom).

A source model constructed solely by co-justified sources will be called a **co-justified model**.

Axiom 10: The axiom of external adoption (in brief, the adoption axiom)
$\forall\varphi(\varphi\neq\tau)\exists\alpha(\alpha\neq i)(i{:}\varphi\leftrightarrow i/{:}\alpha{:}\varphi)$

That is,

$\forall\varphi(\varphi\neq\tau)\exists\alpha(\alpha\neq i)(i/{:}\alpha{:}\varphi)$

Every sentence that the speaking self accepts as true, and that is not an adoption sentence, is transmitted to him or her by some source that he or she has adopted, either fully or partially.

Note 1: This axiom involves only the speaking self and no other sources, since the speaking self in the Source Calculus is not a source in itself (in virtue of the axiom of the speaking self) but the subject to which the sources offer their data. The speaking self is like a judge, while the sources are like evidence. A judge is not evidence, nor does he present evidence, but only accepts it from witnesses and documents, determines how reliable it is, and uses it to draw conclusions.

Note 2: The restriction $\varphi\neq\tau$ ("which is not an adoption sentence") stems from the fact that an adoption sentence constitutes final justification, as mentioned above. While all other types of sentences require justification through identifying their sources, adoption sentences do not require such justification.

Note 3: Since every sentence transmitted by the speaking self has been transmitted to him by a source, the terminology of direct and indirect belief apply to him differently than to other sources. When the speaking self believes a datum by virtue of the fact that he has directly adopted a particular source (i/:a:p), this belief is called **direct belief** (on the part of the speaking self), but when the speaking self believes a datum by virtue of the fact that he has adopted a source that was adopted by another source (i/:a/:b:p), this belief is called **indirect belief** (on the part of the speaking self).

What happens when the sentence transmitted by the speaking self is *non liquet?* Here too we must assume that there is a source transmitting it:

$i{:}(\varphi\backslash\neg\varphi)\leftrightarrow\exists\alpha\ (i/{:}\alpha{:}(\varphi\backslash\neg\varphi))$

In such a situation we assume that there is a given, fixed source that "decides" between the two possibilities, but that the speaking self does not know what it is. We denote this type of unknown source by g'. We can thus state:

The theorem of the undecidedness of the speaking self
$i{:}(\varphi\backslash\neg\varphi)\leftrightarrow i{:}(g'{:}\varphi\vee g'{:}\ \neg\varphi)$

Sometimes the truth source can be **identified** – that is, a constant can be substituted for the variable α – in various ways. Let us illustrate this with a simple, perhaps trivial example:

Given the sentence i:p
$\exists\alpha(\alpha\neq i)(i/{:}\alpha{:}p)$ according to the adoption axiom
Now assume that we have the sentences
a:p
$\neg\exists\alpha(\alpha\neq a)\ \alpha{:}p$
This leads to the conclusion:
$\therefore i/{:}a{:}p$

The theorem on the internality of the justification of adoption:
All adoptions by the speaking self are justified internally, whether directly or indirectly.
Proof:
Assume that the speaking self adopts source a.

1. i/:a
   This adoption has a source, by virtue of the adoption axiom, together with line 1 and modus ponens.
2. $i/a\rightarrow\exists\alpha(\alpha\neq i)(\alpha/{:}a)$
   $\therefore\exists\alpha(\alpha\neq i)(\alpha/{:}a)$
3. The adoption of the source can be either direct or indirect.
   $\forall\alpha(\alpha=a)i/a/{:}a/{:}a\ldots\vee\exists\alpha(\alpha\neq a)i/{:}\alpha{:}a$
4. Direct adoption (the first disjunct) is always internally justified (by definition). In contrast, indirect adoption (the second disjunct) can be justified either internally (i.e. circularly) or externally.
   $\exists\alpha(\alpha\neq a)i/{:}\alpha{:}a\rightarrow(a/{:}\alpha/{:}a\ldots\vee\exists\beta(\beta\neq\alpha)i/{:}\beta/{:}\alpha/{:}a)$
5. The argumentation begun in lines 3–4 about the relation between α and a also applies, *mutatis mutandis*, to the relation between α and β, and so on.
6. However, the chain of adoptions must come to an end (by virtue of the adoption axiom).
7. Therefore the end of the chain of adoptions has to be an internally justified one, whether ordinary circular adoption or self-adoption.

8. Therefore even an indirect source is internally justified, whether directly or indirectly.

QED

Note: This theorem is true for all adoptions by a source, not only by the speaking self.

If these data are a type of "thoughts" (in the broad, Cartesian sense of the word), then the sources, including the division of labor among them, are the factors that determine which thoughts are created. Thus the differences among thoughts stem from the differences among sources and the division of labor among them. When people are involved, this is the phenomenon we sometimes call differences in "ways of thinking" or "approaches", above and beyond the differences in the thoughts themselves.

Nihilistic absurdities

We have now concluded the presentation of the Source Calculus, including its basic terms: sources, the division of labor among them, data, databases and systems. We shall now present three claims that stem from this calculus, which we call "nihilistic absurdities", and which will serve us in later discussions.

Nihilistic absurdities are similar to paradoxes, although not in the narrow sense of the word. These are three claims stemming from the Source Calculus that lead us to skeptical conclusions and thus require a response. They are called "absurdities" because they are reached through *reductio ad absurdum* and seem to be more unacceptable than ordinary paradoxes; they are called "nihilistic" because they go farther than mere skeptical doubts. While skeptical doubts teach us that no belief is justifiable, the nihilistic absurdities teach us than any belief can be justifiable (and hence, not even one is *truly* justifiable).

The first nihilistic absurdity:

Any sentence that does not contradict the laws of logic and the adoption theorems can be justified.

Proof: Given a sentence r.

Given that r is not an adoption sentence: $r \neq t$.

Assume that the speaking self transmits the sentence r (thus both believing it and claiming that it is true): i:r.

Thus there is a source for this belief that was adopted by i: $\exists\alpha(\alpha \neq i)(i/:\alpha:r)$, by the adoption axiom.

α was adopted by the speaking self either directly or indirectly. In either case, this adoption is internally justified (by the theorem of the internality of the justification of adoption).

But internal adoption does not require external justification (by definition).

Therefore any sentence that has any source whatsoever can be justified.

QED

Note: A system, as a logical phenomenon, is closed, and so a sentence transmitted in a given system cannot be refuted or verified by the senses or any other source, however intuitive it might be, unless it is one of the sources on which the system is based. The system's sources verify themselves, and so, within the limits of the system, any datum transmitted by its sources is verified and justified.

Thus, as we can see, any coherentist theory is ruled out (this is in addition to the fact that coherentism too seems to suffer from an infinite regress problem; Sosa 1974). When we take our epistemic system as an information system – one of many possible ones, without the mystification we attach to our ordinary rational system – we should understand that all systems, based on any sources, are equally justifiable. Thus, one can invent a system based on one source which can answer only one single question with yes or no, and, in the terms of Source Calculus, it will be acknowledged as a system just as much as the colossal rational system (a similar example is analyzed below, in Chapter Four). In a coherentist test, the former will certainly be seen as more coherent than the latter, and therefore will be better justified. But a very large number of such systems can be invented for any possible question, they can be invented in a manner ensuring that the desired data are provided, and they will be justified to the same degree under the internal justification of the coherentist test.

This leads us directly to the second nihilistic absurdity:

The speaking self's adoption of a given source is no more justifiable than the adoption of any other source.

Proof: Given two sources a and b.

Assume that the speaking self adopts one source a, so that i/:a.

Now this adoption is internally justified (by the theorem of the internality of the justification of adoption).

Now assume that the speaking self adopts the other system: b, so that i\:b.

This adoption too is internally justified (again, by the theorem of the internality of the justification of adoption).

Thus, the justification for the adoption of both sources is internal.

Therefore, the justification for the adoption of either source is no stronger or weaker than the justification for the adoption of the other one.

QED

Note: This absurdity is already present at the basis of the theorem of the internality of the justification of adoption, and is implicit in the first nihilistic absurdity.

The third nihilistic absurdity:

When the speaking self adopts two sources whose data are contradictory, he or she is not obligated to decide between them.

Proof: Assume that a and b transmit contradictory data:

$a{:}p$
$b{:}\neg p$

Now assume that the speaking self adopts both of them: i/:(a,b).

According to the decision theorem (applied to the given sources)

$$(\neg i{:}\ (p\backslash\neg p))\leftrightarrow(i/{:}(a,b){:}p\vee i/{:}(a,b){:}\neg p)$$

But it is already given that the speaking self does not decide to accept one of these data:

$\neg i/{:}(a,b){:}p$
$\neg i/{:}(a,b){:}\neg p$

Therefore the consequent is negated.

As a result, the antecedent is refuted as well (by the truth table of the biconditional).

Therefore $i{:}(p\backslash\neg p)$

QED

Note: If this absurdity were the only one, it would not be an absurdity at all. After the two previous absurdities, however, it implies that the speaking self can find his way out of the first two absurdities by refraining from having any beliefs about the world, and this situation may be conceived to be just as acceptable as the adoption of a source that provides data for beliefs about the world. Put bluntly, this absurdity implies that a person who adopts a particular source has no advantage over one who chooses not to think at all.

The nihilistic absurdities stem directly from the Source Calculus. We must therefore try to see how it we can overcome them, if this is at all possible.